EE 2240

Problem #11

For the system described by $\ddot{x}+12\dot{x}+40x=0$:

a. Determine the characteristic equation.

$r^2+12r+40=0$

b. Determine the natural frequency, ω_n.

$\omega_n^2=40 \Rightarrow \omega_n=\sqrt{40}=2\sqrt{10} \approx 6.325$

c. Determine the damping ratio, ζ.

$2\zeta\omega_n=12 \Rightarrow \zeta=\frac{12}{2\omega_n}=\frac{3}{\sqrt{10}} \approx 0.949$

d. Determine the numerical values of the two roots of the characteristic equation.

$r^2+12r+40=(r+6)^2+2^2$

$\Rightarrow r=-6 \pm j2$

e. Classify the system as *overdamped*, *critically damped*, *underdamped*, or *undamped*.

The system is underdamped $(0<\zeta<1)$

f. Assuming $x(0)=-1$ and $\dot{x}(0)=8$, determine the solution of the given equation.

$x(t)=e^{-6t}(K_1\cos 2t+K_2\sin 2t)$

$\dot{x}(t)=-6e^{-6t}(K_1\cos 2t+K_2\sin 2t)$
$+e^{-6t}(-2K_1\sin 2t+2K_2\cos 2t)$

$x(0)=K_1=-1$
$\dot{x}(0)=-6K_1+2K_2=8$
$\Rightarrow K_1=-1,\ K_2=1$

$\therefore x(t)=e^{-6t}(-\cos 2t+\sin 2t), \quad t \geq 0$